1.0 Background and Theory

Amplitude fading in a general multipath environment may follow different distributions depending precisely on the area covered by measurements that is the presence or absence of a dominating strong component, and some other conditions.

The model behind Rician fading is similar to that for Rayleigh fading except that in Rician fading includes a strong dominant component, which for instance can be the line-of-sight wave.

Rayleigh fading used for paths with no dominant signal is derived from Clarke [1] studies. In 1968 Clarke has presented theoretical model that statistically describes the behavior of small scale fading in urban multipath environments. In this model the arriving multipath components are confined to the horizontal plane. The phases of these multipath components are assumed to be uniformly distributed over [0-2\pi].

The Rice distribution proposed by Rice [2] in 1944 is a fading model is suitable for representing the small scale fluctuations of the signal envelope in a narrowband channel where a LOS exists between the transmitter and the receiver terminals such as in microcellular or a satellite channels. In other words, the effect of a dominant signal arriving with many weaker scatter signals (Rayleigh fading) gives rise to the Rician distribution. As the dominant signal becomes weaker, the composite signal resembles a noise signal, which has an envelope that is described by Rayleigh distribution. Thus, the Rician distribution degenerates to a Rayleigh distribution when the dominant component fades away. This is shown in the next formulation section.

The accepted hidden assumption behind the Rician model [2] is that the numerous individual constituent propagation paths of both the in-phase component and the quadrature phase component of the received signal are seen as jointly Gaussian random variables, i.e. the distribution of the sum of a large number of random variables is a normal distribution that can completely described by its mean and variance. This known as the Central-Limit-Theorem and it justifies the use random variables for many engineering applications.

In the case of a Rayleigh fading channel, the in-phase and quadrature phase components will both have zero-means, however in the case of a Rician fading, the mean value of (at least) one component is non-zero due to the strong constant component in the received signal.
2.0 PDF Formulation

Let \( R = X + jY \) be complex Gaussian with real and imaginary components that are independent, in the polar coordinate representation that \( X = R \cos \theta \) and \( Y = R \sin \theta \).

As mentioned formally, now assume that \( X \) and \( Y \) have equal variances \( \sigma^2 \). While the imaginary component \( Y \) has a zero mean \( m_q \) the real component \( X \) which will represent the dominant signal, has a positive mean \( m_i \).

The pdf of the Rician distribution is widely given by [3]:

\[
P_{\text{RICE}}(r) = \frac{r}{\sigma^2} e^{-\frac{r^2 + s^2}{2\sigma^2}} I_0 \left( \frac{rs}{\sigma^2} \right)
\]

\( r > 0, \ s > 0, \ \sigma > 0 \)

Where \( r \) is the amplitude of the received envelope, and the parameter \( s^2 \) is the so called non-centrality parameter computed from the first moment and is given by:

\[
s^2 = \sum_{i=1}^{n} (m_i)^2
\]

\( I_0 \) is the zero\(^{th} \) modified Bessel function of first kind.

On the contrary, as \( s \to 0, \ I_0(0) = 1 \) yields the Rayleigh distribution given by:

\[
P_{\text{RAYLEIGH}}(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}
\]

3.0 Rician \( K \) Factor

According to [2] [3] [4] and elsewhere in the literature, \( s^2/2 \) dominates the coherent (LOS path) and \( \sigma^2 \) dominates the incoherent power (indirect paths). The Rician \( K \)-factor is physically defined as the ratio of signal power in dominant component over the (local-mean) scattered power. Thus it is given in dB by:

\[
K = 10 \log_{10} \left( \frac{s^2}{2\sigma^2} \right)
\]
Knowledge of the Rician K Factor can be useful in determining the signal strengths i.e. SIR, bit error rate, link budget calculation and capacity of a channel among other useful metrics. It also is important in describing the distribution of the density functions in a wireless communication channel.

Given (1) we can now write the PDF of the Rician distribution as function of $K$ as follows:

$$P_{RICE}(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} e^{-Kr} I_0\left(\frac{r}{\sigma}\sqrt{2K}\right) \quad (5)$$

Therefore it can be clearly concluded if there is no dominant signal in the T-R propagation path, $K=0$, $I_0(0)=1$ and $e^{-K}=1$ producing a Rayleigh PDF.

### 4.0 CDF Formulation

Scanning the literature, a number of attempts have been given for approximating the cumulative distribution function for the Rician distribution. In [3] the Rician CDF takes the shape of:

$$C_{RICE}(r) = 1 - e^{\left(-\frac{K+\frac{r^2}{2\sigma^2}}{2}\right)} \sum_{n=0}^{\infty} \left(\frac{\sigma\sqrt{2K}}{r}\right)^n I_n \left(r\sqrt{2K}/\sigma\right) \quad (6)$$

Clearly this formula is more difficult to evaluate than the PDF of (3) due to the summation of an infinite number of terms, requiring double or quadruple precision, which preferably is avoided in numerical evaluations. Nonetheless [3] has shown a range of CDF evaluated from the above equation. On the other hand, in practical terms it can be sufficient to increase $n$ to a certain value such as 50 terms that could be sufficient enough to reduce the remaining terms contribution to a negligible level [8].

Other alternatives forms have also been addressed in the literature. In [5] the CDF is expressed as:

$$C_{RICE}(r) = 1 - Q\left(\frac{\sqrt{2K}}{\sigma}, \frac{r\sqrt{2}}{\sigma}\right) \quad (7)$$

Where $Q(a,b)$ is the Marcum’s Q function and $a & b$ are non-negative real numbers.

Derivation of (7) is given in [6] & [7]. Recently another approach has also been presented in [9], based on non-linear tabulated coefficients, however it is similar in a way to the previous ones in the published literature.
5.0 Extracting the Rician K Factor from Measurements

Recall the K factor represents the ratio of the received LOS component to the received scattered components. This concept of K factor, first put forth in [19], is very difficult to measure in a physically meaningful manner; i.e., by isolating the direct signal from the scattered components. From the available literature, a number of approaches are available for extracting the Rician K factor from a measured time-series data; these are summarized in the following sections.

5.1 Method of Moments

Traditional methods of estimating the Rician K factor from the measured received power versus time such as the Maximum Likelihood method are relatively impractical and time consuming as we will see in the following sections.

Given the Rician distribution it is possible to obtain its moments from the measured data, and as a result it can then be possible to estimate a locally constant signal s from magnitude data points r.

The well-known authors in [10] have presented a very significant moment method of estimating the Rician K factor. The method is simple and rapid approach where the K factor is an exact function of the moments estimated from the time series data. The method seems to be reliable and widely used in the technical literature by a number of professional people such as Prof. Theodore Rappaport in his work given in [4] as part of the IEEE 802.16 Broadband Wireless Access Working Group project.

The method simply applies the definition of K factor given in (4) which requires the estimation of the Rician parameters s (mean) and σ (variance). These can be calculated from the mean and standard deviation of the measured data, conversely it is set by finding the first two moments of the measured data.

Using the same notation in [4], the first moment is the time averaged amplitude of the measured data, i.e. its theoretical mean value \( \mu_p \). The second moment of interest is \( \sigma_p \) the local RMS fluctuation of the received signal about \( \mu_p \).

It is worth nothing that the standard method for using the local RMS value for normalization is proposed by Clarke and given in [3] and is defined as:

\[
RMS = \left( \frac{1}{n} \sum_{i=1}^{n} r(x_i)^2 \right)^{\frac{1}{2}}
\]  

(8)

The definitions of the first and second moments [4] are given by:
\[ \mu_p = \frac{s^2}{2} + \sigma^2 \quad (9) \]

\[ \sigma_p = \sqrt{s^4 + s^2 \sigma^2} \quad (10) \]

Note that \( \sigma \) here the standard deviation differs from \( \sigma_p \), the 2\( \text{nd} \) momentum of the measured data.

By using the above definitions and subsequently by simply solving for Rician parameters \( s \) and \( \sigma \), the Rician \( K \) factor (in dB) is now given by:

\[ K = 10 \log_{10} \left( \frac{s^2}{2\sigma^2} \right) = 10 \log_{10} \left( \frac{\sqrt{\mu_p^2 - \sigma_p^2}}{\mu_p - \sqrt{\mu_p^2 - \sigma_p^2}} \right) \quad (11) \]

This leads to the same derivation given in [10], which also contains the validation of the proposed moment method.

### 5.2 Graphical Moment Method

Another fine method for extracting the \( K \) factor is given in [11]. This method is very much similar to the moment method described formally, and is given by the following expression:

\[ \frac{E[r]}{\sqrt{E[r^2]}} = \sqrt{\frac{\pi}{4K+1}} \exp \left( -\frac{K}{2} \right) \left( (K+1)J_0 \left( \frac{K}{2} \right) + KI_1 \left( \frac{K}{2} \right) \right) \quad (12) \]

Where \( E[r] \) is the average amplitude defined by \( \mu_p \) (1\( \text{st} \) moment) earlier in this report, and \( E[r^2] \) is the average of the squared amplitude defined by \( \sigma_p \) (2\( \text{nd} \) moment) earlier in this report. The following figure shows equation (10) graphically. Therefore from the ratio of the 1\( \text{st} \) moment to the 2\( \text{nd} \), the \( K \) factor can be estimated from the graph in Fig.1.
In [11] the $K$ values generated from the above expression are checked optimally by plotting the CDF obtained empirically from the measured data and the theoretical CDF. They also have used the Kolmogorov-Smirnov test [3] in order to best fit the data. As a matter of interest, the Kolmogorov-Smirnov test is a distribution fitting algorithm that provides robust, but nevertheless, computationally complex procedure which is not easy to implement online. For curiosity, a $K$ factor mean value of 10.52dB and 9.13dB was
obtained for their indoor measurements in a university common room and a workshop, respectively.

5.3 Second and Forth Moment Method

Another moment-based method very similar to the graphical method is proposed by [12], which aims on providing a closed-form expression for extracting $K$ factor. The method develops from the graphical method by the use of the second and forth moments of the measured data. The motivation behind this method corresponds to finding the exact value for $K$ to solve equation (12), which involves a complex inverting numerical procedure.

$$f_{2,4}(K) = \frac{(K+1)^2}{K^2 + 4K + 2}$$

(13)

Where $f_{2,4}$ is a function of $K$ and refers to the ratio of the second moment to the forth of the measured data. Clearly, calculating the inverse of the above equation involves finding the roots of a second-order polynomial which can be done in a closed form, and is straightforwardly given by:

$$K_{2,4} = \frac{-2\mu_2^2 + \mu_4 + \mu_2 \sqrt{2\mu_2^2 - \mu_4}}{\mu_2^2 - \mu_4}$$

(14)

Note that notation of the second moment here $\mu_2$ is the same as $\sigma_p$ defined formerly.

5.4 Maximum Likelihood Estimation

The authors in [13] have constructed an optimum maximum likelihood estimator for the Rician distributed data. The maximum estimator of $s$ is defined as the estimator maximizing $L$ or $\log L$ as a function of $s$. Where $L$ is the joined PDF of a sample of $n$ independent observations $r_i$. This is called the likelihood function of the measured sample and is given by:

$$L = \prod_{i=1}^{n} P_{RICE}(r, s)$$

(15)

Hence using equation (1) in the above equation gives:

$$\log L \approx \sum_{i=1}^{n} \log I_o \left( \frac{S r_i}{S^2} \right) - \sum_{i=1}^{n} \frac{S^2}{2\sigma^2}$$

(16)

The maximum likelihood is therefore the global maximum of $\log L$:
From the above, finding the optimum value for $s$ is not direct and requires finding the maximum of $\log L$ which in general will be an exhaustive iterative numerical process. It will be only necessary at certain instances [13] to utilize this method in which is out of our scope.

Moreover the authors of [14] have studied the statistical performances of the moment-based method as a less complex alternative to the maximum likelihood method. The asymptotic analysis given reveals that both methods are equally efficient, but, however the maximum likelihood is unsuitable for real-world applications.

### 6.0 True Values for the Rician $K$ Factor

From measurements our research group has conducted and elsewhere in the literature, antennas with narrow beamwidths seem to be more beneficial to obtain and model a high value of $K$ as possible. This directly implies isolating the direct path by limiting the Rayleigh fading that occurs from multipath powers.

Now since the $K$ factor can serve as a metric for differentiating Rician from Rayleigh distributions, the question becomes, “how large must $K$ be to truly have a Rician distribution?” The possible answer to this question is given from measurements published literature, below are a couple of them.

Measurements done by Theodore Rappaport [4] and his colleagues where conducted at 38GHz utilizing a transmitter horn antenna of 19dBi and a receiver parabolic antenna of 39dBi gain. The T-R separation is 265m and is a partly obstructed (mostly by tress) LOS link. The purpose of the measurements was to evaluate the $K$ factor at different rain conditions as part of the IEEE 802.16 Broadband Wireless Access Working Group project.

The $K$ factor was extracted using the method of moments. According to the authors, the value of the $K$ factor was surprisingly as high as 8dB at the rain rate of 220mm/hr and 17dB at dry conditions.

Another set of measurements where conducted at 900MHz by John Davies [15] in 1994 at the University of California. One tremendous purpose of the measurements was to determine the Rician parameters in vehicle to vehicle RF propagation. The values of $K$ factors obtained were between 6-18dB. Its worth noting out here that in this particular interesting measurements, despite the high values of $K$, the transmitter and receivers where of same heights. However at some rare instants they experienced low values of $K$ factors (~1.38dB) which does is not of any concern as the author had explained. As long as a Rician channel actually exists with a dominant component of at least two paths; a
LOS and a reflection of the roadway; these two components may interact with each other in such a way to have a sinusoidal type of frequency that might affect the computations.

Furthermore data presented in [16] and again from the IEEE 802.16 Broadband Wireless Access Working Group, it was suggested that the median value for the Rician $K$ factor is in the range from 5-20dB. According to [17] values of the Rician $K$ factor in indoor and outdoor systems can in general be as high as 25dB.

Finally, all the $K$ values excavated from the above presented measurements agree totally with the general rule of thumb given by [18] which quotes that a near LOS condition is associated with a Rician $K$ Factor of 10dB or more.

7.0 Implementation and Evaluation

Now that the parameter extraction methods have been presented, this section implements the Moment Method and the Graphical Method. The data used in this example are the industrial estate link time-series measurements conducted on Sep 05. Fig.2 shows the results obtained from the moment method.
Moreover, the Graphical Method produces the following values for the ratio of 1\textsuperscript{st} moment over the 2\textsuperscript{nd} moment given in eq.12; 0.9473, 0.9584, 0.9615, and 0.9836 which from Fig.1 refer to $K$ factors of 5.44, 6.53, 7.78 and 11.46dB for antenna gains of 10, 15, 20 and 29dBi respectively. In comparison, the values generated from this method and the ones obtained directly from the Moment Method are relatively close, despite the slight differences which can be due to exact plotting errors.

Furthermore, to evaluate the accuracy of the $K$ Factors values obtained, a Rician distribution can be fitted to match the empirical measured PDF. Fig.3 shows two examples for the $K$ Factors of 5.96 and 14.64dB obtained from the Moment Method at a receiver antenna gain of 10 and 29dBi respectively.

As the wavelength at 40GHz is only 7.5mm, the received signal will experience great fluctuation over small space variations of the receiver’s terminals; therefore the values of the $K$ factor will strongly depend on the receiver’s precise location. This comes to an extreme level when the T-R separation is small and when there is a large number of multipaths at the receiver’s vicinity which represents a typical microcell environment. The measurement conducted in our campus is one very good example. It came to my attention, at some particular instant, a very low value for the $K$ Factor has been observed, at around -1.75dB, this occurred while moving the 10dBi receiver antenna in a direction perpendicular to the link. In this extreme case the empirical PDF distribution of the measured data reasonably follows a Rayleigh distribution and has been fairly fitted with a -1.75dB $K$ Factor Rician distribution. On the other side, very high $K$ Factors values occur at some instances, such as 15dB. Exclusively for our measurements, where the receiver is intentionally moved once in the direction of the link and another in perpendicular direction; it would seem more appropriate to take the mean $K$ value between the both.
Hence, for the industrial link, these values become 7.63, 8.24, 8.57 and 12.31 for antenna gains of 10, 15, 20 and 29dBi respectively.

8.0 Summary

From the literature it seems that the Moment Method has been widely used and is the most appropriate technique for extracting the Rician parameters. The justification of the Graphical Moment Method and The Second and Fourth Moment Method use the same basis for the derivations and refers to a number of published papers not available to me at the moment. However it has been shown that the Graphical method yields relatively similar values to the Moment method.

Additionally, it has been shown that one, very explicit method for accomplishing the evaluation of these methods is to match various Rician distribution functions to the tabulated results from the measurements. By varying the Rician parameters, the distribution that most closely matches is optimum.

9.0 References


